# **Control strategies of the Limited Bandwidth Hydro-pneumatic Active Suspension for Road Vehicle**

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Abstract— The active suspension required energy input levels and the high component costs. In this work, the Limited bandwidth hydro-pneumatic active suspension (LBA) is used and it has all the advantages of active system except that the actuator is limited at 6 Hz bandwidth, thus making it economical on power consumption and cost. The aim of this study is to develop four degrees of freedom (DOF) half vehicle model including the LBA suspension system. The LQR and Fuzzy Logic Control (FLC) are used to evaluate the vehicle ride performance for LBA suspension. The result indicated that the LBA suspension with LQR gives better ride performance compared with the passive suspension system. On the other hand, the FLC improved the vehicle ride performance in terms of front and rear body acceleration by 4% and 7.5% respectively compared with LBA with LQR. The power demand for LBA with LQR and FLC are evaluated and discussed.

*Index Terms*— Limited bandwidth hydropneumatic active suspension, Fuzzy Logic Control (FLC), LQR, Anti-Lock Braking System (ABS)

## I. INTRODUCTION

The main idea of LBA suspension is to utilize the active device to control the system dynamics around the body resonance and to also allow passive elements to exercise suitable control for the higher frequency components. The active suspension systems offer the best overall performance, but are considerably impractical, because of extremely high cost involved. The LBA suspension is more practical and performs nearly as well [1-3]. Other researcher [4] developed a methodology for the design and evaluation of a slow-active vehicle suspension system. They designed an optimal multivariable controller for a full car model in terms of seven degrees-of-freedom. This controller requires a linear quadratic regulator form with supplementary states to add integral action. Their results showed that the slow-active systems offer significant improvements in controlling body resonances. This system consumes low power compared to active systems. This fact is recorded in the vehicle literature over the past few years [5].

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Several control strategies for active and slow active are used [6-9]. Recently, a Model Reference Control (MRC) strategy for active suspension System is developed [10]. The MRC technique utilized both wheelbase preview concepts and suspension look-ahead preview and the MRC methodology depended on an ideal hybrid skyhook-ground hook concept. MRC technique utilized 8 PID controller for each wheels and body control. The proposed MRC strategy with controller PID was able to track the performance of an ideal hybrid skyhookground-hook system and provided a significant improvements in road holding and ride comfort together. It is clear from the previous literature that more investigations of LBA suspension are required. In this work, the LQR and Fuzzy Logic Control (FLC) are used to evaluate the vehicle ride performance for LBA suspension (LBA). Also, the power demand for LBA with LQR and FLC are evaluated and discussed.

#### II. VEHICLE MATHEMATICAL MODEL

#### A. Equation of motion

The four degrees of freedom half vehicle model used is shown in Fig. 1.

The equations of motion can be derived by applying Newton's second law for the vehicle body and wheel masses respectively as follow [1, 11, 13]:

$$M_b \ddot{Z}_b = -F_{sf} - F_{sr} \tag{1}$$

$$I_b \ddot{\theta}_b = L_f F_{sf} - L_r F_{sr} - h_f (F_{xf} - F_{rf}) - h_r (F_{xr} - F_{rr})$$
(2)

$$M_{wf}\ddot{Z}_{wf} = -K_{tf}(Z_{wf} - Z_{of}) + F_{sf}$$
(3)

$$M_{wf}\ddot{Z}_{wr} = -K_{tr}(Z_{wr} - Z_{or}) + F_{sr}$$
<sup>(4)</sup>

Where,

$$h_f = H + (Z_{bf} - Z_{wf}) + (Z_{wf} - Z_{of})$$
(5)

$$h_r = H + (Z_{br} - Z_{wr}) + (Z_{wr} - Z_{or})$$
(6)

For passive suspension system the suspension dynamic forces can be written as the following:

$$F_{sf} = K_{sf} \left( Z_{bf} - Z_{wf} \right) + C_{sf} \left( \dot{Z}_{bf} - \dot{Z}_{wf} \right)$$
(7)

$$F_{sr} = K_{sr}(Z_{br} - Z_{wr}) + C_{sf}(\dot{Z}_{br} - \dot{Z}_{wr})$$
(8)



Fig. 1 Half vehicle model with LBA suspension system

(1)	Oil tank	(6,7)	Font and rear gas springs
(2)	Pump	(8,9)	Front and rear throttle
			valves
(3)	Main accumulator	(10,11)	Front and rear suspension
			struts
(4,5)	Front and rear direction control valves		

Where:

Zb	Body	vertical	dis	placement	at	center	of	gravity
								0

Z<sub>bfr</sub> Front and rear body vertical displacement

Z<sub>of,r</sub> Front and rear road input

 $\mathbf{Z}_{wf,r}$  Front and rear wheel vertical displacement

 $\boldsymbol{\theta}$  Body pitch angle

The LBA suspension operates to control the vehicle ride characteristics over the lower frequency range in particular up to 6 Hz. For LBA suspension system the dynamic forces of the suspension can be written as the following:

$$F_{sf} = A_{st} \left[ P_{gstf} (V_{gstf}^{\gamma} / ((V_{gstf} - V_{gf})^{\gamma}) - P_{gstf} + C_1 A_{st} (\dot{Z}_{bf} - \dot{Z}_{wf}) \right]$$
(9)

$$F_{sr} = A_{st} \left[ P_{gstr} (V_{gstr}^{\gamma} / ((V_{gstr} - V_g)^{\gamma}) - P_{gstr} + C_1 A_{st} (\dot{Z}_{br} - \dot{Z}_{wr}) \right]$$
(10)

Most of the studies proved that, the automobile model can be reduced in the 2-dimensional model seen in Figure 1 as long as the automobile speed is constant. It can be observed, the coherence between the right and left tracks is likely to be high and the road surface can be considered as a cylindrical for long wavelengths. so, the 2 sides of the automobile will behave in the similar fashion. Also, nothing which the motions excited in the automobile would mostly involve wheel hop for short wavelengths. A Little body motion will happen, and right and left would interact very little [11, 12].

### B. Road input and vehicle parameters

The road input is presented using the following equation;

$$\dot{R}_o(t) = -2\pi f_o R_o(t) + \sqrt{G \cdot G_o(t) \cdot v(t)} w(t)$$
(11)

In this study,  $R_o$ , Go, v, w(t) and  $f_o$  are the displacement of road input, the road input roughness coefficient, the driving speed, zero-mean Gaussian white noise, with its intensity1 and the low cut-off frequency (0.01 Hz). The road roughness coefficients and the half vehicle parameters used for the calculations are shown in Table I and Table II respectively [12].

TABLE I				
ROAD ROUGHNESS COEFFICIENT				
Road roughness	Class			
-				
256*10-6	В			
512*10-6	С			
	TABLE I ROAD ROUGHNESS CC Road roughness 256*10 <sup>-6</sup> 512*10 <sup>-6</sup>			

TABLE II VEHICLE PARAMETERS

Parameter	Value/Unit	Description
M <sub>b</sub>	690 kg	Body mass
$M_{wf}$ , $M_{wr}$	40.5 and 45 kg	Front and rear wheel masses
Ip	1222 kg. m <sup>2</sup>	Body moment of inertia
K <sub>sf</sub> , K <sub>sr</sub>	17 and 22 KN/m	Front and rear spring stiffness
C <sub>sf</sub> , C <sub>sr</sub>	1.5 KN.s/m	Front and rear damping coefficient
K <sub>tf</sub> , K <sub>tr</sub>	192 KN/m	Front and rear tire stiffness
Н	0.328	The height of vehicle C.G from
		the road surface
L <sub>f</sub> , L <sub>r</sub>	1.25 and 1.51 m	Distance from C.G to front and rear
		axles
γ	1.4	Gas constant
A <sub>st</sub>	$8.04 * 10^{-4} m^2$	Strut area

#### **III. CONTROLLERS**

Two different control algorithms have been advanced to the LBA suspension system. The first strategy is based on optimal control theory using limited state feedback concept, while the second strategy is related to FLC. In order to design the LBA controllers, it is assumed that the vehicle has body and wheel vertical acceleration sensors at each corner to support the control algorithm with the body and wheel vertical accelerations states. Also, the vehicle has a suspension travel sensor at each corner to support the suspension travel states.

## A. Optimal Control theory

Optimal control theory that interested in operating the dynamic system at least cost. The theory is a part of applied mathematics which has apriority to get the control law for applying it on the dynamical system through time period until optimizing the objective function. A set of linear differential equations use to describe system dynamics. In addition to, it has various applications in both engineering and science. For instance, the dynamical system can be an automobile with controls related to vertical automobile dynamics, and the objective of that may be to improve ride comfort with minimum power demand and maintain SWS. Also, possible that dynamical system may be spacecraft, with its objective to reach moon with the controls related to rocket thrusters [11]. Optimal control can be seen as a control strategy in control theory. One of the major results in this theory is which the solution has been provided by the linear quadratic regulator.

As known, applying the full state feedback control concept for the LQR controllers is unpractical due to the difficulties in measuring the road input. Therefore, the limited state feedback concept is selected to derive the feedback LQR control law shown in equation (12) [10].

$$P_{gdi} = K_{1i}\ddot{Z}_{bi} + K_{2i}\ddot{Z}_{wi} + K_{3i}(Z_{bi} - Z_{wi})$$
(12)

Where  $K_{1..3}$ , are the LQR control gains.

### B. Fuzzy Logic Control

Fuzzy-Logic is used to design a practical and cost-effective controller for the LBA suspension system. FLC is considered as one of the smartest control methods and it also, presents different unparalleled features which make from FLC is a best choice for many control issues. Non-linear system which is impossible to represented by mathematically can be controlled by FLC. It also does not need precise or noise free input. It can be programmed to control the system even if a feedback sensor is damaged. The control output is a smooth function in spite of a wide extent of input variations. so, any sensor data that supplies many indications of any systems actions and reactions are suitable. For all of that, FLC let the sensors to be inexpensive and inaccurate so this keeps the overall system cost and intricacy low. Because of these advantages, the Fuzzy-Logic control is used to develop a practical and cost effective the LBA suspension system controller. The FLC controller requires the body vertical acceleration and the suspension velocity signals as a controller inputs, which leads to cost improvement in the overall system as the wheel vertical acceleration signals/sensors not required. The output signals are the demand  $P_{qd}$  at each corner. The rule base and interface engine are formed with Mamdani-Type of fuzzy inference, while the defuzzification process is based on center of area method. The rule base of the developed FLC algorithm is shown in table III.

TABLE III Slow-active Flc rule base

P <sub>gd</sub>			From	it or Rea	r Suspen	sion Velo	ocity	
		NB	NM	NS	ZE	PS	PM	PB
ion	NB	NB	NB	NB	NB	NB	NB	NB
elerat	NM	NM	NM	NM	NM	NM	NM	NM
/ Aco	NS	NS	NS	NS	NS	NS	NS	NS
Body	ZE	ZE	ZE	ZE	ZE	ZE	ZE	ZE
Rear	PS	PS	PS	PS	PS	PS	PS	PS
int or	PM	PM	PM	PM	PM	PM	PM	
Frc	PB	PB	PB	PB	PB	PB	PB	PB

The direction control valve is considered as the first order transfer function shown in Equation 13 has been used for simulating the dynamics of slow active control valve

$$Q_{Ai}(s) = \frac{1}{(t_d(s)+1)} Q_{AD_i}(s)$$
(13)

Where,  $Q_{AD_i}(s)$  is the Laplace transform of the desired flow rate  $Q_{AD_i}(t)$ , while  $Q_{Ai}(s)$  is the actual flow rate whose time domain form is  $Q_{Ai}(t)$ , and  $t_d(s)$  is the Laplace transform of the time delay constant. The following performance index shown in Equation 14 is selected for the optimization process.

$$J = q_1 RMS_{Acc_f} + q_2 RMS_{Acc_r} + q_3 RMS_{DTL_f} + q_4 RMS_{DTL_r} + q_5 RMS_{SWS_f} + q_6 RMS_{SWS_r}$$
(14)

As shown in equation (14), the performance index is the weighted sum of the Root Mean Square (RMS) the dynamic tire loads, body vertical accelerations and the suspension deflections. Moreover, the performance index components have been normalized regards to passive suspension and have been weighted by  $(q_{1.6})$  the weighting parameters in order to confirm the importance of all component [12].

#### IV. RESULTS AND DISCUSSIONS

A. Comparison between passive suspension, slow active with optimal control and FLC using road input (class B)

In the LBA suspension an actuator with 6Hz bandwidth is used to control the suspension. The vehicle has been simulated over a road input (Class B) with constant vehicle speed 100 km/hr. Fig. 2 and Fig. 3 show the power spectral density of the body accelerations, suspension working space and dynamic tyre load for the passive suspension system, LBA active suspension systems with LQR and FLC. The comparisons are made in terms of power spectral density. In Fig. 2 shows the body bounce, pitch accelerations, vehicle body at CG and rear body acceleration for the passive system and LBA suspension with LQR and FLC. It can be noticed that the ride performance of the LBA suspension with FLC gives worthwhile improvements than the passive suspension system in terms of DTL and body acceleration. Moreover, the system gives better ride performance than the LBA suspension with LQR. These improvements are clearly seen in the frequency range up to 6.0 Hz.

Fig. 3 shows the rear SWS and DTL for the passive system and LBA suspension with LQR and FLC. It can be seen that two point were emerge; (i) the rear SWS and DTL of the LBA suspension with FLC gives better improvements than the passive suspension system and LBA suspension with LQR; (ii) there is improvements in terms of dynamic tyre load around the unsprung mass resonance frequency is observed with both LBA suspension system with FLC and LQR.

The summary of ride performance improvements in terms of root means square of front and rear body accelerations using road input roughness (Class (B)) is shown in Table IV. The percentages reduction of front and rear body accelerations for LBA suspension with LQR compared with passive suspension are 12.3% and 16.1% respectively. On the other hand, the percentages reductions of front and rear body accelerations for LBA suspension with FLC compared the same system with LQR are 4% and 7.5% respectively. Furthermore, this percentage reduction for the same system is increased to 16.6% in case of pitch acceleration.

TABLE IV

System		Passive	LBA with	LBA with
Performance		Suspension	LQR	FLC
Freed	Acc, m/sec <sup>2</sup>	0.5849	0.5132	0.4925
FIOII	SWS, m	0.00556	0.006459	0.006996
	DTL, N	404.6	430.5	448.6
D	Acc, m/sec²	0.7507	0.6299	0.5824
Rear	SWS, m	0.005936	0.006346	0.006729
	DTL, N	419.4	454.2	488.2
CG Body, Acc. m/sec <sup>2</sup>		0.6515	0.5618	0.5298
Pitch Acc., $m/sec^2$		0.09763	0.06549	0.05465



Fig. 2. Power spectral density of body C.G bounce, pitch accelerations and rear body acceleration for passive and LBA suspension with LQR and FLC using road input (class B)



Fig. 3. Power spectral density of rear SWS and DTL for passive and LBA suspension with LQR and FLC using road input (class B)

# B. Comparison between passive suspension, slow active with optimal control and FLC using road input (class C)

Fig. 4. shows comparison of the body bounce, rear body acceleration and pitch accelerations at the vehicle body CG for the passive system, LBA suspension systems with LQR and FLC in terms of power spectral densities. The power spectral densities curves showed a clear improvement around body resonance peak in the body CG, pitch and rear body accelerations of the LBA suspension system with FLC in comparison with a passive suspension system and the LBA suspension system with LQR, in the overall frequency range. Also, a very small improvement around the unsprung mass resonance peak is observed for both LBA suspension system with FLC and LQR.



Fig. 4. Power spectral density of body C.G bounce, pitch accelerations and rear body acceleration for passive and LBA suspension with LQR and FLC using road input (class B)

Fig. 5 shows the rear SWS and DTL for the passive system and LBA suspension with LQR and FLC using road input (Class C). It can be seen that, there are clear improvements around body and wheel resonances peaks for rear DTL.



Fig. 5. Power spectral density of rear SWS and DTL for passive and LBA suspension with LQR and FLC using road input (class C)

The summary of vehicle ride performance in terms of root means square of front and rear body, accelerations, pitch and vehicle body CG using road input roughness (Class (C)) is shown in Table V. The percentage improvements of pitch, front, rear body accelerations for LBA suspension with LQR compared with passive suspension are 33%, 12.3% and 16.1% respectively. On the other hand, the percentages reductions of pitch, front and rear body accelerations for LBA active suspension with FLC compared the same system with LQR are 16.6%, 4% and 7.5% respectively.

Overall, the LBA suspension with FLC gives a significant ride performance improvement compared with passive suspension system and LBW with LQR for both roads. The percentages improvements for LBA suspension with FLC compared with the same system with LQR are constant for both roads (Class B and C). The values of body accelerations are depended on the type of road input, vehicle speed and the control strategy used.

		TABLE	/	
System	E PERFORMANCE	Dassivo	I BA with	LASS C)
System		rassive	LDA with	LDA
Perform	nance	Suspension	LQR	with
				FLC
Front	Acc.	0.8272	0.8242	0.6969
	m/sec <sup>2</sup>			
	SWS, m	0.007863	0.009138	0.009861
	DTL, N	572.2	608.8	633.5
Rear	Acc,	1.062	0.8907	0.8342
	m/sec <sup>2</sup>			
	SWS, m	0.008395	0.008978	0.009616
	DTL, N	593.2	642.2	682
CG Body Acc.		0.9213	0.7945	0.753
m/sec <sup>2</sup>				
Pitch Acc. $m/sec^2$		0.1381	0.09254	0.08545

C. Comparison of slow active suspension with optimal control and FLC in terms of power requirements

Mean power demand for LBA suspensions with LQR and FLC are calculated in Table\_VI. Looking first at the power demand results showed that there are little differences between these systems. The mean power demand of LBA suspension for rear with LQR and FLC at vehicle speed of 100 km/hr, are 45 W and 44.95 W respectively. It can be seen that, although LBA suspensions with FLC gives better ride performance compared with the same system with LQR, it is required nearly the same power demand for actuator. So, it is more suitable to be used in the LBA suspensions.

TABLE VI					
RIDE PER	FORMANCE IM	PROVEMENTS WITH	I ROAD INPUT	Г (CLASS C)	
Vehicle	Power R.M.S. of LBA Power R.M.S. of LBA			.M.S. of LBA	
speed	suspension with LQR		suspension with FLC		
(km/hr)	(1	watt)	(watt)		
80	Front	35.27	Front	35.25	
80	Rear	36.77	Rear	36.71	
100	Front	44.09	Front	44.05	
100	Rear	45.01	Rear	44.95	
120	Front	52.91	Front	52.87	
120	Rear	53.83	Rear	53.7	

Comparison between LBA suspension with LQR and FLC at front and rear in terms of power demand using vehicle speed of 100 km/hr is shown in Fig. 6. It can be seen that, although the mean values are low, the peak values are much higher in comparison with the mean value. For more clarity, time sections for the comparison between LBA suspension with LQR and FLC at front and rear in terms of power demand is presented in Fig. 7.



Fig. 6 Comparison between LBA suspension with LQR and FLC at front and rear in terms of power demand



Fig. 7. More clarity comparison between LBA suspension with LQR and FLC at front and rear in terms of power demand

# V. CONCLUSION

1. The LBA suspension with LQR gives better ride performance compared with the passive suspension system for both roads used. The proposed control

law is based on the optimal linear control theory; the system uses the more practical limited state feedback law

- 2. The LBA suspensions with FLC gives better ride performance compared with the same system with LQR, and it is required nearly the same power demand for actuator. So, it is more suitable to be used in the limited bandwidth hydro-pneumatic active suspensions.
- The percentages reductions of pitch, front and rear body accelerations for LBA suspension with FLC compared the same system with LQR are 16.6%, 4% and 7.5% respectively.

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# Notation

Symbol	Definition
$A_{st}$	Strut area
$C_{I}$	Throttle valve constant
$C_d$	Vehicle aerodynamic coefficient
FLC	Fuzzy logic control
F.	Front and rear passive and LBA
<b>I</b> ' sf,r	suspension forces
$F_{rf,r}$	Front and rear rolling resistance force
$F_{xf,r}$	Front and rear brake force
$F_{zf,r}$	Front and rear normal force
J	Performance index
Η	The height of C.G from road surface
$I_b$	Vehicle body moment of inertia
K <sub>sf,r</sub>	Front and rear spring stiffness
K <sub>tf,r</sub>	Front and rear tire stiffness
$K_{1-3}$	The LQR control gains
L	Vehicle wheel base
L <sub>f,r</sub>	Distance from C.G to front and rear axles
$M_b$	Vehicle body mass
$M_{wf}$	Front wheel mass
$M_{wr}$	Rear wheel mass
$M_t$	Total vehicle mass
$P_{gd}$	Demand signal pressure
$P_{gsti}$	Front and rear gas spring static pressure

$Q_{Ai}$	Front and rear actual flow rate
$Q_{AD}$	Front and rear desired flow rate
$q_{1-6}$	Cost function weighting parameters
<i>RMS<sub>ACCi</sub></i>	Front and rear body acceleration root mean square
<i>RMS</i> <sub>SWSi</sub>	Front and rear Suspension Working Space root mean square
RMS <sub>DTLi</sub>	Front and rear Dynamic Tire Load root mean square
V <sub>gsti</sub>	Front and rear gas spring static volume
$Z_b$	Body vertical displacement at center of gravity
$Z_{bi}$	Front and rear body vertical displacement
Z <sub>of,r</sub>	Front and rear road input
Z <sub>wf,r</sub>	Front and rear wheel vertical displacement
$\ddot{ heta}$	Body pitch acceleration